HDZ-003-016304 Seat No. ________

M. Sc. (Sem. III) (CBCS) Examination

November / December – 2017

Mathematics : CMT-3004

(Discrete Mathematics)

(Old Course)

Faculty Code : 003
Subject Code : 016304

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) There are five questions.

(2) All questions are compulsory.

(3) Each question carries 14 marks.

1 Fill in the blanks : (Each question carries two marks) **14**

(1) Let A be a non empty set. Define the concept of the free semi group generated by A.

(2) Define the concept of regular expressions over a set A.

(3) Define a complemented lattice and illustrate it with an example.

(4) Define a modular Lattice. Illustrate that a finite lattice need not be a modular.

(5) Define : (i) a conditional statement and (ii) a Moore machine.

(6) Define machine congruence on a finite state machine.

(7) State Kleene's theorem.

2 Attempt any **two** : **14**

(a) Let R be a congruence relation defined on a semi group (S, *). Describe in detail the construction of the quotient semi group of S determined by R. Let A be a non empty set. Show that $(\mathbb{N} \cup \{0\},+)$ is isomorphic to a quotient semi group of A*. **7**

(b) Let $n \geq 1$ and let $f: B_n \rightarrow B$. Prove that f is produced by a Boolean expression. **7**

(c) State and prove the fundamental theorem of homomorphism of semi groups. **7**

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3 All compulsory : **14**

(a) Let V be a vector space over a field F. Show that the lattice of subspaces of V is modular. **5**

(b) Let $(L, \leq)$ be a Boolean Algebra. Let $a, b \in L$. Show that (i) $(a \vee b)' = a' \wedge b'$ and (ii) $(a \wedge b)' = a' \vee b'$. **5**

(c) Prove that there exists no semi group homomorphism from $(\mathbb{N}, x)$ onto $(5\ \mathbb{N}, x)$. **4**

OR

3 All compulsory : **14**

(a) Let R be a symmetric relation defined on a non empty set A. Prove that $\mathbb{R}^{\infty}$ is symmetric. **4**

(b) Let$(L, \leq)$ be a finite Boolean Algebra. Let $a \in L$, $a \neq 0$. Let $\{c_1, c_2, ... c_k\}$ be the set of all atoms c of L such that $c \leq a$. Prove that $a = V_{i=1}^{k} ci$. **5**

(c) Let $f: B4 \rightarrow B$ be such that S(f) = {0000, 0001, 0011, 0010, 1000, 1001, 1111}. Construct the Karnaugh map of f and find a Boolean expression for the function f. **5**

4 Attempt any **two** : **14**

(a) Let $(L, \leq)$ be a finite Boolean Algebra. Prove that the number of atoms of $(L, \leq)$ is equal to the number of coatoms of $(L, \leq)$ **7**

(b) Let M = (S, I, $\mathcal{F}$, s_0, T) be a moore machine. If R is the w – compatibility relation defined on S, then show that R is a machine congruence on M and L (M)=L(M/R). **7**

(c) Let $(L, \leq)$ be a lattice, If $(L, \leq)$ is not modular, then prove that $(L, \leq)$ will contain a sublattice M which is isomorphic to the pentagon lattice. **7**

5 Attempt any **two** : **14**

(a) State and prove Pumping lemma. **7**

(b) Let (L, *) be a commutative semi group in which a*a=a for a$\in$L. Prove that the relation $\leq$ defined on L by a$\leq$b if and only if a*b=b is a partial order and for any a, b$\in$L, a*b is the least upper bound of {a, b} in (L, $\leq$). **7**

(c) For the languages given in (i) and (ii) below, construct a phrase structure grammar G such that L(G) = L. **7**

(i) $L = \{a^n b^m / n \geq 1, m \geq 3\}$ and

(ii) $L = \{x^n y^m / n \geq 2, m \geq 0 \text{ and even}\}$

(d) Let P be a proportional function. Prove **7**

(i) $\sim (\forall x P(x)) \equiv \exists x(\sim P(x))$ and

(ii) $\sim (\exists x P(x)) \equiv \forall x(\sim P(x))$.